TOPOLOGY MIDTERM EXAMINATION

You can quote any result proved in class (unless you are being asked to prove it). Total marks: 35

(1) Let $\mathbb{R}^\omega$ be the countably infinite product of $\mathbb{R}$ with itself, and let $\mathbb{R}^\infty$ be the subset of $\mathbb{R}^\omega$ consisting of sequences which are eventually zero, that is, $(a_i)_{i=1}^\infty$ such that only finitely many a_i's are nonzero. Determine the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\omega$ with respect to the box topology, the uniform topology, and the product topology on $\mathbb{R}^\omega$. (3 x 3 = 9 marks)

(2) Consider $\mathbb{Z}$ as a normal subgroup of the additive group $\mathbb{R}$ of real numbers. Prove that the group $\mathbb{R}/\mathbb{Z}$ is isomorphic to the group S^1 as topological groups. (5 marks)

(3) Give an example (with details) of a connected topological space which has infinitely many path connected components. If X is a locally path connected topological space and U is a connected open subset of X, then prove that U is path connected. (3+3 = 6 marks)

(4) Let I_o^2 be the oredered square. Is I_o^2 locally path connected? locally connected? compact? (3 x 3 = 9 marks)

(5) State and prove the Lebesgue number lemma. (2+4 = 6 marks)

Date: February 25, 2015.